Duplex Encoding of Staircase At-Most-One Constraints for the Antibandwidth Problem

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logics LOGICAL METHODS IN COMPUTER SCIENCE

FWF

THE ANTIBANDWIDTH PROBLEM

The Antibandwidth Problem (ABP)

- Graph labelling max-min optimization problem

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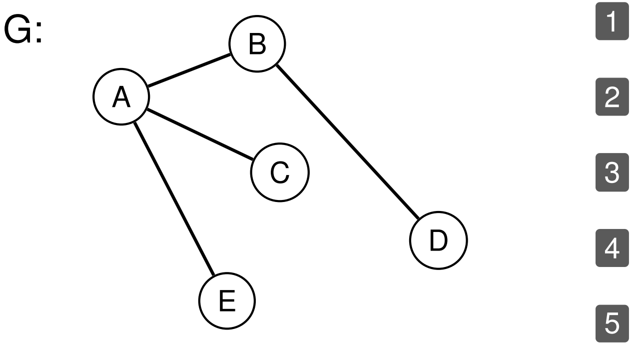

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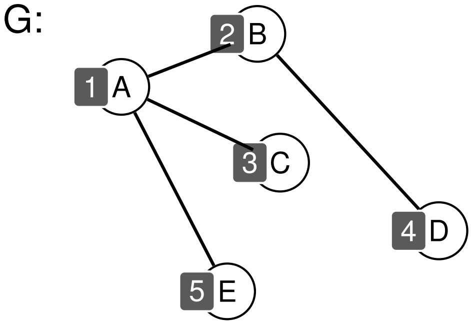
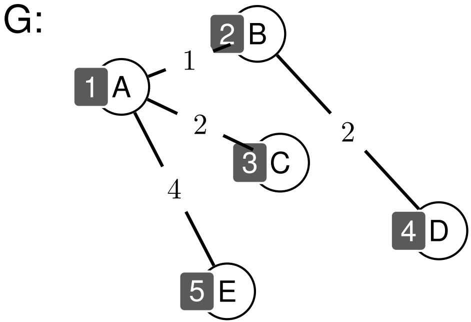


- Antibandwidth: smallest difference of connected labels

The Antibandwidth Problem (ABP)

- Graph labelling max-min optimization problem



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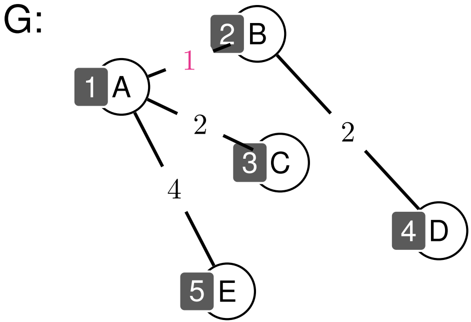

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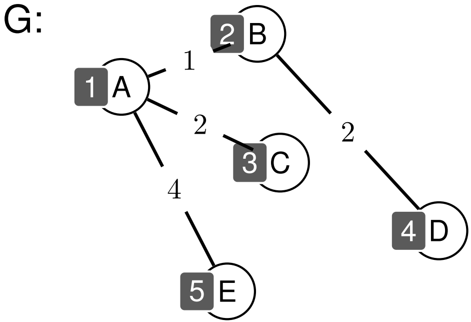
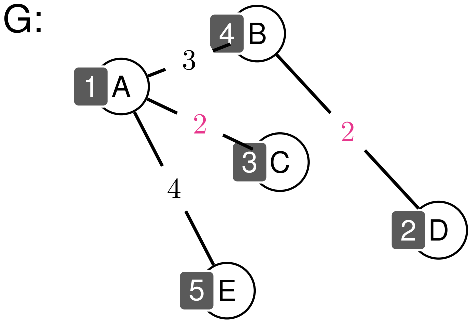


- Antibandwidth: smallest difference of connected labels
- Goal: Find a labelling with the highest possible antibandwidth

The Antibandwidth Problem (ABP)

- Graph labelling max-min optimization problem



- Antibandwidth: smallest difference of connected labels
- Goal: Find a labelling with the highest possible antibandwidth
- Radio frequency assignment, multiprocessor scheduling

RELATED WORK

Iterative MIP Formalization of ABP [Markus Sinnl - CEJOR'20]

Is there a labelling of G with antibandwidth $> k$?

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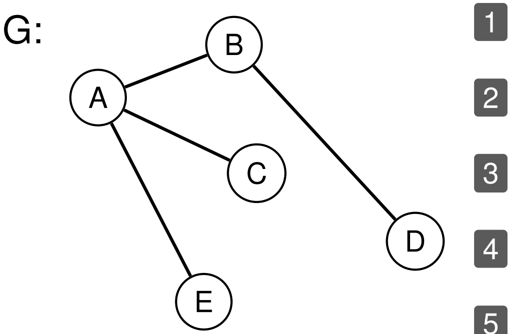

Iterative MIP Formalization of ABP [Markus Sinnl - CEJOR'20]

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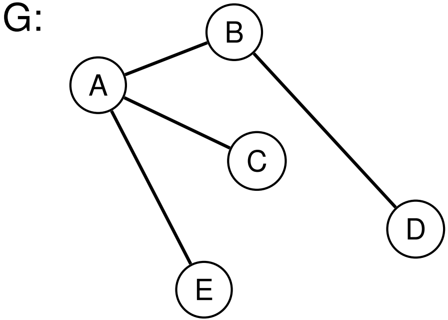

1	x_A^1	x_B^1	x_C^1	x_D^1	x_E^1
2	x_A^2	x_B^2	x_C^2	x_D^2	x_E^2
3	x_A^3	x_B^3	x_C^3	x_D^3	x_E^3
4	x_A^4	x_B^4	x_C^4	x_D^4	x_E^4
5	x_A^5	x_B^5	x_C^5	x_D^5	x_E^5

$x_v^\ell \Leftrightarrow$ label ℓ is assigned to vertex v

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Iterative MIP Formalization of ABP [Markus Sinnl - CEJOR'20]

Is there a **labelling** of G with antibandwidth $> k$?

- Each label is assigned to exactly one node (Exactly-One constraints)

$$\sum_{i \in V} x_i^\ell = 1 \quad \forall \ell \in \{1, \ldots, |V|\}$$

Iterative MIP Formalization of ABP [Markus Sinnl - CEJOR'20]

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- Each node gets exactly one label assigned (Exactly-One constraints)

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Iterative MIP Formalization of ABP [Markus Sinnl - CEJOR'20]

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- Each node gets exactly one label assigned (Exactly-One constraints)

$$\sum_{\ell \in \{1,\ldots,|V|\}} x_i^\ell = 1 \quad \forall i \in V$$

- No edge connects labels from any k-long range (At-Most-One constraints)

$$\sum_{\lambda \leq \ell \leq \lambda+k} (x_i^\ell + x_{i'}^\ell) \leq 1 \qquad \forall \{i, i'\} \in E,\ 1 \leq \lambda \leq |V| - k$$

ITERATIVE SAT FORMALIZATION OF ABP

Iterative SAT Formalization of ABP

$$\forall \{i, i'\} \in E : \forall 1 \leq \lambda \leq |V| - k : \sum_{\lambda \leq \ell \leq \lambda + k} (x_i^\ell + x_{i'}^\ell) \leq 1$$

Iterative SAT Formalization of ABP

$$\forall \{i, i'\} \in E : \forall 1 \leq \lambda \leq |V| - k : \sum_{\lambda \leq \ell \leq \lambda + k} (x_i^\ell + x_{i'}^\ell) \leq 1 \qquad \equiv$$

$$\forall \{i, i'\} \in E : \bigwedge_{\lambda=1}^{(|V|-k)} \left(\sum_{\ell=\lambda}^{(\lambda+k)} x_i^\ell + x_{i'}^\ell \leq 1 \right)$$

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Iterative SAT Formalization of ABP

$$\forall \{i, i'\} \in E : \forall 1 \leq \lambda \leq |V| - k : \sum_{\lambda \leq \ell \leq \lambda+k} (x_i^\ell + x_{i'}^\ell) \leq 1 \qquad \equiv$$

$$\forall \{i, i'\} \in E : \bigwedge_{\lambda=1}^{(|V|-k)} \left(\sum_{\ell=\lambda}^{(\lambda+k)} x_i^\ell + x_{i'}^\ell \leq 1 \right) \qquad \equiv$$

$$\forall \{i, i'\} \in E : \bigwedge_{\lambda=1}^{(|V|-k)} \left(\sum_{\ell=\lambda}^{(\lambda+k)} x_i^\ell \leq 1 \wedge \sum_{\ell=\lambda}^{(\lambda+k)} x_{i'}^\ell \leq 1 \wedge \left(\sum_{\ell=\lambda}^{(\lambda+k)} x_i^\ell \leq 0 \vee \sum_{\ell=\lambda}^{(\lambda+k)} x_{i'}^\ell \leq 0 \right) \right)$$

Iterative SAT Formalization of ABP

$$\forall \{i,i'\} \in E : \forall 1 \leq \lambda \leq |V| - k : \sum_{\lambda \leq \ell \leq \lambda+k} (x_i^\ell + x_{i'}^\ell) \leq 1 \qquad \equiv$$

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- Decomposed into “staircase” at-most-one and at-most-zero constraint sets

SAT ENCODING OF STAIRCASE AT-MOST-ONE CONSTRAINT SETS

Staircase At-Most-One Constraint Sets

$$\text{SCAMO}(X, w) = \bigwedge_{i=0}^{(|X|-w)} \left(\sum_{\ell=i+1}^{(i+w)} x^{\ell} \leq 1 \right)$$

Staircase At-Most-One Constraint Sets

$$\mathrm{SCAMO}(X, w) = \bigwedge_{i=0}^{(|X|-w)} \left(\sum_{\ell=i+1}^{(i+w)} x^{\ell} \leq 1 \right) = \mathrm{SEQ}(0, 1, w, X, \{1\})$$

Staircase At-Most-One Constraint Sets

$$x^1 + x^2 + x^3 + x^4 + x^5 \leq 1$$
$$x^2 + x^3 + x^4 + x^5 + x^6 \leq 1$$
$$x^3 + x^4 + x^5 + x^6 + x^7 \leq 1$$
$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1$$
$$x^5 + x^6 + x^7 + x^8 + x^9 \leq 1$$
$$x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1$$
$$x^7 + x^8 + x^9 + x^{10} + x^{11} \leq 1$$
$$x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1$$
$$x^9 + x^{10} + x^{11} + x^{12} + x^{13} \leq 1$$
$$x^{10} + x^{11} + x^{12} + x^{13} + x^{14} \leq 1$$
$$x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1$$

$$\mathrm{SCAMO}(X, w)$$
$$X = \langle x^1 \cdots x^{15} \rangle$$
$$w = 5$$

Staircase At-Most-One Constraint Sets

$$
\begin{aligned}
&x^1 + x^2 + x^3 + x^4 + x^5 \leq 1 \\
&x^2 + x^3 + x^4 + x^5 + x^6 \leq 1 \\
&x^3 + x^4 + x^5 + x^6 + x^7 \leq 1 \\
&x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \\
&x^5 + x^6 + x^7 + x^8 + x^9 \leq 1 \\
&x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1 \\
&x^7 + x^8 + x^9 + x^{10} + x^{11} \leq 1 \\
&x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1 \\
&x^9 + x^{10} + x^{11} + x^{12} + x^{13} \leq 1 \\
&x^{10} + x^{11} + x^{12} + x^{13} + x^{14} \leq 1 \\
&x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1
\end{aligned}
$$

$$\mathrm{SCAMO}(X, w)$$

$$X = \langle x^1 \cdots x^{15} \rangle$$

$$w = 5$$

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Encoding of Staircase At-Most-One Constraint Sets

Is there a truth assignment to the variables of X such that each at-most-one constraint of $\mathrm{SCAMO}(X, w)$ is satisfied?

$$
\begin{aligned}
x^1 + x^2 + x^3 + x^4 + x^5 &\leq 1 \\
x^2 + x^3 + x^4 + x^5 + x^6 &\leq 1 \\
x^3 + x^4 + x^5 + x^6 + x^7 &\leq 1 \\
x^4 + x^5 + x^6 + x^7 + x^8 &\leq 1 \\
x^5 + x^6 + x^7 + x^8 + x^9 &\leq 1 \\
x^6 + x^7 + x^8 + x^9 + x^{10} &\leq 1 \\
x^7 + x^8 + x^9 + x^{10} + x^{11} &\leq 1 \\
x^8 + x^9 + x^{10} + x^{11} + x^{12} &\leq 1 \\
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x^{10} + x^{11} + x^{12} + x^{13} + x^{14} &\leq 1 \\
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\end{aligned}
$$

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Off-the-shelf SAT encoding of 11 constraints

Encoding of Staircase At-Most-One Constraint Sets

Is there a truth assignment to the variables of X such that each at-most-one constraint of $\mathrm{SCAMO}(X, w)$ is satisfied?

$$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1}$$
$$x^2 + x^3 + x^4 + x^5 + x^6 \leq 1$$
$$x^3 + x^4 + x^5 + x^6 + x^7 \leq 1$$
$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1$$
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$$\boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}$$

Duplex encoding of 3 constraints

Encoding of Staircase At-Most-One Constraint Sets

$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1}$ $\boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1}$ $\boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}$

Encoding of Staircase At-Most-One Constraint Sets

$x^1 + x^2 + x^3 + x^4 + x^5 \leq 1$ $x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1$ $x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1$

- Every other constraints of the SCAMO are decomposed:
  $x_1 + \cdots + x_n \leq 1 \equiv$

  $(x_1 + \ldots + x_i \leq 1) \wedge (x_{i+1} + \ldots + x_n \leq 1) \wedge (x_1 + \ldots + x_i \leq 0 \vee x_{i+1} + \ldots + x_n \leq 0)$

Encoding of Staircase At-Most-One Constraint Sets

$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \equiv$$

$$x^4 + x^5 \leq 1 \land x^6 + x^7 + x^8 \leq 1 \land ((x^4 + x^5 \leq 0) \lor (x^6 + x^7 + x^8 \leq 0))$$

$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1}$ $\boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1}$ $\boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}$

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Encoding of Staircase At-Most-One Constraint Sets

$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \equiv$$

$$x^4 + x^5 \leq 1 \land x^6 + x^7 + x^8 \leq 1 \land ((x^4 + x^5 \leq 0) \lor (x^6 + x^7 + x^8 \leq 0))$$

$$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1} \quad \boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1} \quad \boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}$$

$$((x^8 + x^9 + x^{10} \leq 0) \lor (x^{11} + x^{12} \leq 0)) \land x^8 + x^9 + x^{10} \leq 1 \land x^{11} + x^{12} \leq 1$$

$$\equiv x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1$$

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Encoding of Staircase At-Most-One Constraint Sets

$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \equiv$$

$$x^4 + x^5 \leq 1 \wedge x^6 + x^7 + x^8 \leq 1 \wedge ((x^4 + x^5 \leq 0) \vee (x^6 + x^7 + x^8 \leq 0))$$

$$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1} \quad \boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1} \quad \boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}$$

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$$\equiv x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1$$

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- Duplex: consider both left-associative and right-associative addition

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$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \equiv$$

$$x^4 + x^5 \leq 1 \wedge x^6 + x^7 + x^8 \leq 1 \wedge ((x^4 + x^5 \leq 0) \vee (x^6 + x^7 + x^8 \leq 0))$$

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$$\equiv x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1$$

- Every other constraints of the SCAMO are decomposed:
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- Duplex: consider both left-associative and right-associative addition
- At-most-one and at-most-zero constraints over the necessary sub-sums

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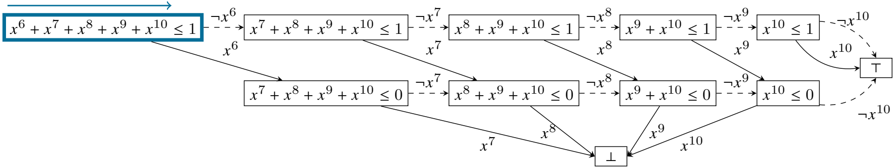
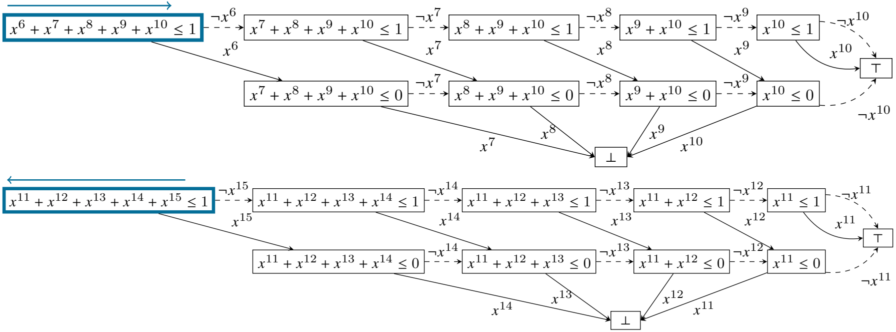
SAT Encoding with Binary Decision Diagrams



SAT Encoding with Binary Decision Diagrams

- Considered associativity of addition determines variable order in BDD



SAT Encoding with Binary Decision Diagrams

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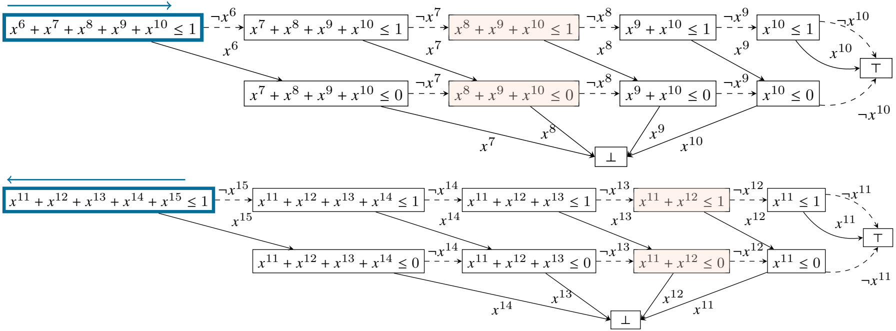
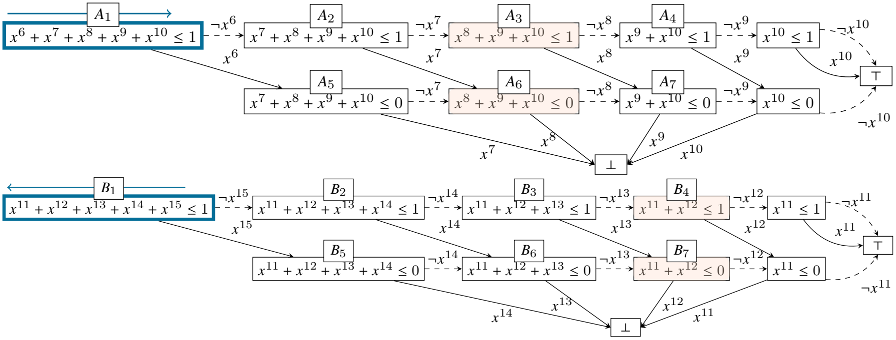


$$x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1 \equiv x^8 + x^9 + x^{10} \leq 1 \wedge x^{11} + x^{12} \leq 1 \wedge \left((x^8 + x^9 + x^{10} \leq 0) \vee (x^{11} + x^{12} \leq 0)\right)$$

JKU

SAT Encoding with Binary Decision Diagrams

- Considered associativity of addition determines variable order in BDD



$$x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1 \equiv x^8 + x^9 + x^{10} \leq 1 \land x^{11} + x^{12} \leq 1 \land \Big((x^8 + x^9 + x^{10} \leq 0) \lor (x^{11} + x^{12} \leq 0)\Big)$$

$$\equiv A_3 \land B_4 \land (A_6 \lor B_7)$$

SAT Encoding of Staircase At-Most-One Constraint Sets

Is there a truth assignment to the variables of X such that each at-most-one constraint of $\mathrm{SCAMO}(X, w)$ is satisfied?

$$
\begin{array}{l}
\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1} \\
x^2 + x^3 + x^4 + x^5 + x^6 \leq 1 \\
x^3 + x^4 + x^5 + x^6 + x^7 \leq 1 \\
x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \\
x^5 + x^6 + x^7 + x^8 + x^9 \leq 1 \\
\boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1} \\
x^7 + x^8 + x^9 + x^{10} + x^{11} \leq 1 \\
x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1 \\
x^9 + x^{10} + x^{11} + x^{12} + x^{13} \leq 1 \\
x^{10} + x^{11} + x^{12} + x^{13} + x^{14} \leq 1 \\
\boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1}
\end{array}
$$

SAT Encoding of Staircase At-Most-One Constraint Sets

Is there a truth assignment to the variables of X such that each at-most-one constraint of $\mathrm{SCAMO}(X, w)$ is satisfied?

$$\boxed{x^1 + x^2 + x^3 + x^4 + x^5 \leq 1} \quad \longrightarrow \text{Duplex – BDD}$$
$$x^2 + x^3 + x^4 + x^5 + x^6 \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^3 + x^4 + x^5 + x^6 + x^7 \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^4 + x^5 + x^6 + x^7 + x^8 \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^5 + x^6 + x^7 + x^8 + x^9 \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$\boxed{x^6 + x^7 + x^8 + x^9 + x^{10} \leq 1} \quad \longrightarrow \text{Duplex – BDDs}$$
$$x^7 + x^8 + x^9 + x^{10} + x^{11} \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^8 + x^9 + x^{10} + x^{11} + x^{12} \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^9 + x^{10} + x^{11} + x^{12} + x^{13} \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$x^{10} + x^{11} + x^{12} + x^{13} + x^{14} \leq 1 \quad \longrightarrow 3 \text{ clauses}$$
$$\boxed{x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \leq 1} \quad \longrightarrow \text{Duplex – BDD}$$

EXPERIMENTS

Duplex Encoding of SCAMOs – SAT Experiments

- Linear sized SAT encoding of staircase at-most-one constraint sets

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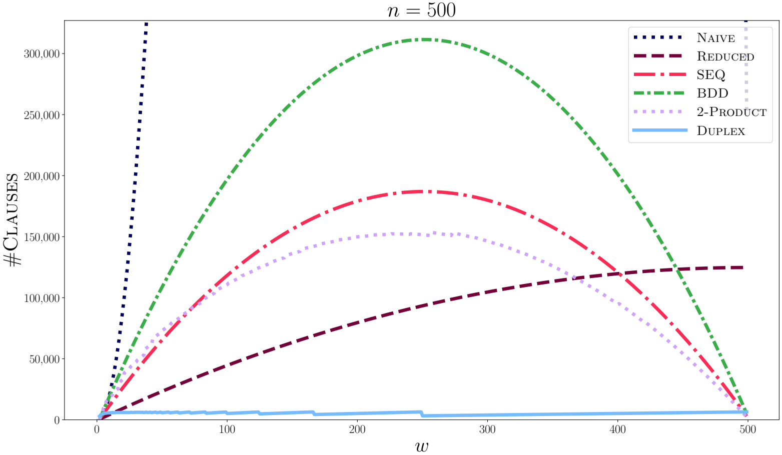

Duplex Encoding of SCAMOs – ABP Experiments

- Harwell-Boeing Sparse Matrix Collection (smaller instances)

Instance	$\|V\|$	$\|E\|$	LB	UB	SAT – Duplex			MIP – $F_e(\mathrm{k})$			CP – CPLEX			CP – MZ-Chuffed		
					Obj.	Time	MB	Obj.	Time	MB	Obj.	Time	MB	Obj.	Time	MB
A-pores_1	30	103	6	8	**6**	185.52	52	**6**	23.71	29	6-8	TO	57	**6**	5.97	11
B-ibm32	32	90	9	9	**9**	1.30	11	**9**	28.57	29	**9**	7.35	20	**9**	17.4	11
C-bcspwr01	39	46	16	17	**17**	3.85	13	**17**	6.64	28	**17**	18.78	21	17	TO	11
D-bcsstk01	48	176	8	9	**9**	0.25	14	**9**	62.28	36	**9**	20.15	21	**9**	6.35	12
E-bcspwr02	49	59	21	22	**21**	3.37	13	**21**	774.02	205	**21**	22.84	19	**21**	673.44	11
F-curtis54	54	124	12	13	**13**	1.33	18	**13**	12.56	32	**13**	34.66	21	**13**	2.14	11
G-will57	57	127	12	14	**13**	0.57	19	**13**	15.4	33	**13**	44.75	21	**13**	2.69	11
H-impcol_b	59	281	8	8	**8**	0.54	22	**8**	0.47	24	8-22	TO	63	**8**	23.3	12
I-ash85	85	219	19	27	**23**	TO	331	20	TO	133	22-31	TO	37	21	TO	12
J-nos4	100	247	32	40	**35**	585.33	190	-	TO	106	34-47	TO	31	-	TO	12
K-dwt__234	117	162	46	58	49	TO	477	48	TO	264	**51-57**	TO	33	-	TO	11
L-bcspwr03	118	179	39	39	**39**	0.99	58	**39**	0.52	21	**39**	110.92	22	**39**	26.42	12

On a cluster with Intel Xeon E5-2620 v4 @ 2.10GHz CPUs, TO = 1800 seconds. Used SAT solver in Duplex is CaDiCaL 1.2.1. Solvers $F_e(k)$ and CP-CPLEX are as-is from [Markus Sinnl - CEJOR'20] and CP-MZ-Chuffed is via MiniZincIDE-2.3.2.

Duplex Encoding of SCAMOs – ABP Experiments

- Harwell-Boeing Sparse Matrix Collection (larger instances)

Instance	$\|V\|$	$\|E\|$	LB	UB	SAT – Duplex			MIP – $F_e(k)$			CP – CPLEX			CP – MZ-Chuffed		
					Obj.	Time	MB	Obj.	Time	MB	Obj.	Time	MB	Obj.	Time	MB
M-bcsstk06	420	3720	28	72	**34**	TO	1621	33	TO	625	-	TO	20	-	TO	35
N-bcsstk07	420	3720	28	72	**34**	TO	1621	33	TO	634	-	TO	20	-	TO	35
O-impcol_d	425	1267	91	173	**99**	TO	1043	95	TO	691	-	TO	18	-	TO	24
P-can__445	445	1682	78	120	-	TO	1581	-	TO	644	-	TO	18	-	TO	24
Q-494_bus	494	586	219	246	-	TO	1167	**220**	TO	905	-	TO	18	-	TO	21
R-dwt__503	503	2762	46	71	**62**	TO	1680	52	TO	911	-	TO	19	-	TO	31
S-sherman4	546	1341	256	272	-	TO	1129	-	TO	1033	-	TO	19	-	TO	24
T-dwt__592	592	2256	103	150	-	TO	2253	-	TO	1068	-	TO	20	-	TO	37
U-662_bus	662	906	219	220	**220**	319.73	1564	-	TO	1320	-	TO	19	-	TO	28
V-nos6	675	1290	326	337	-	TO	1571	-	TO	1434	-	TO	20	-	TO	28
W-685_bus	685	1282	136	136	**136**	14.33	1428	**136**	9.24	37	-	TO	20	-	TO	29
X-can__715	715	2975	112	142	-	TO	3312	-	TO	1468	-	TO	21	-	TO	39

On a cluster with Intel Xeon E5-2620 v4 @ 2.10GHz CPUs, TO = 1800 seconds. Used SAT solver in Duplex is CaDiCaL 1.2.1. Solvers $F_e(k)$ and CP-CPLEX are as-is from [Markus Sinnl - CEJOR'20] and CP-MZ-Chuffed is via MiniZincIDE-2.3.2.

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