Incremental Proofs for Bounded Model Checking

Methoden und Beschreibungssprachen zur Modellierung
und Verifikaton von Schaltungen und Systemen
February 15, 2024

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Outline

Preliminaries & Motivation

Our Contributions

Conclusion

Boolean Satisfiability Problem (SAT)

- Propositional logic

$$(a \vee \overline{b}) \wedge (a \vee b) \wedge (\overline{a} \vee \overline{b})$$

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$$\{a = \top, b = \bot\}$$

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F_i satisfiable $\leftrightarrow$ there is a property violation up to i steps

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- In practice problems often formulated step-by-step:

Is formula F_0	satisfiable?
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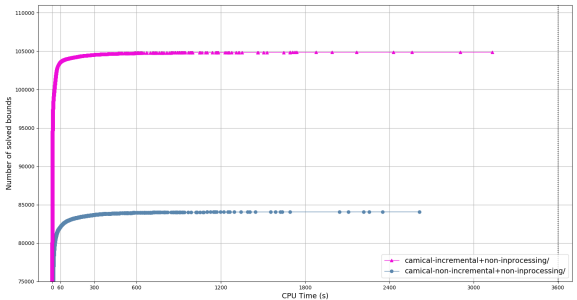
- Sequence of decision problems where each problem is an extension or slight modification of the previous one.
- Assumptions: Temporary constraints that are considered in the next query and after that immediately deleted.
- **Incremental Solvers**: Can solve each formula with the exact same solver.
  - \+ Reuse reasoning steps instead of repeating them.

SAT-based Bounded Model Checking & Incremental SAT

+ Incremental reasoning can lead to significant speed up in BMC



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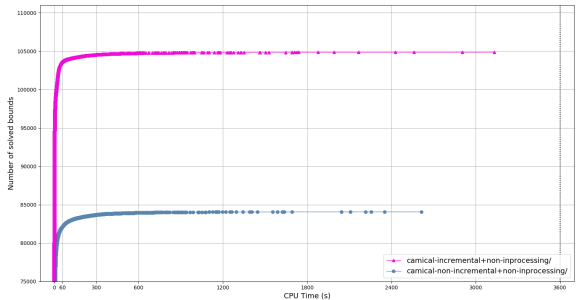

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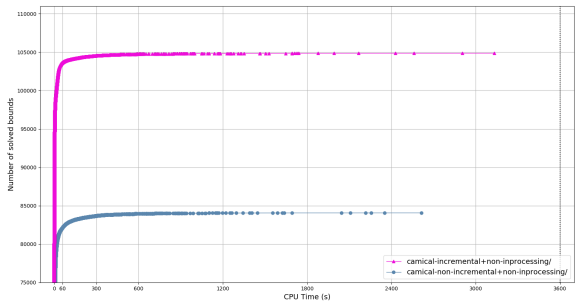

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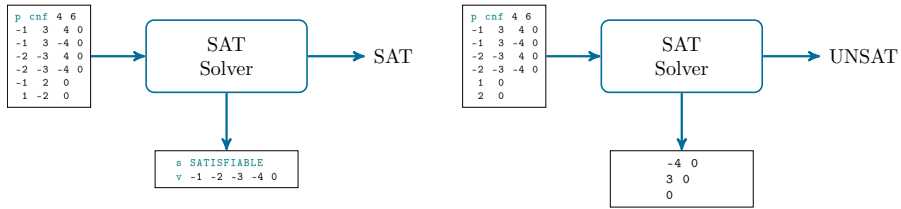
− **Not all results are certified in incremental SAT solvers.**

Verifiable Results – Proofs & Solutions of SAT Solvers



- Standardized input and output formats, guaranteed verifiable certificates.

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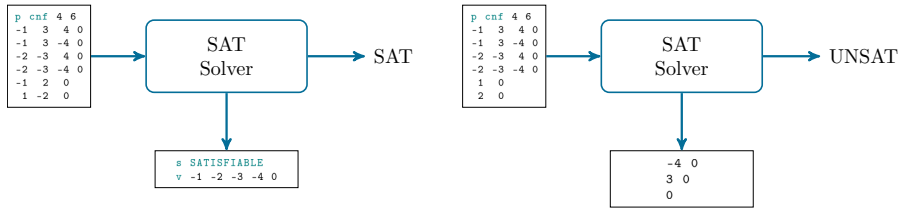

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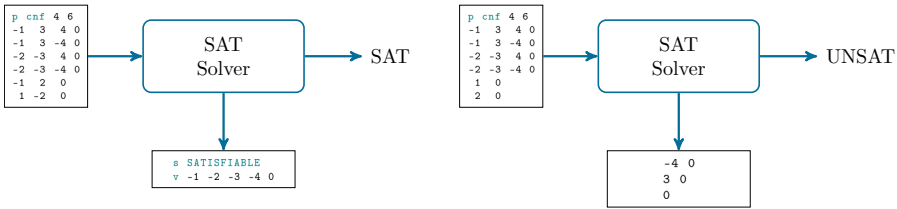

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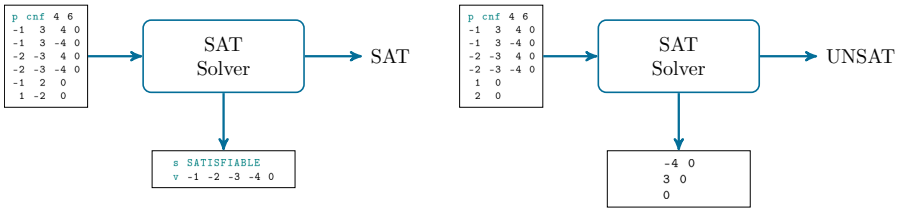

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  - with assumptions: **Not defined, no guaranties what will be derived.**

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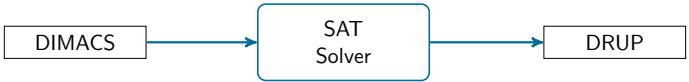

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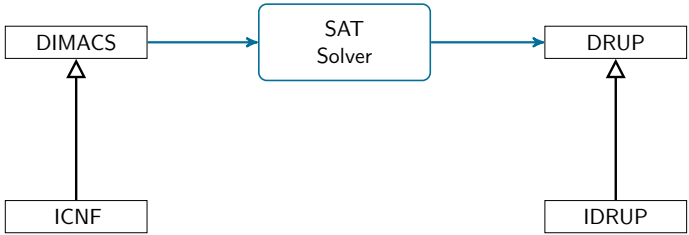

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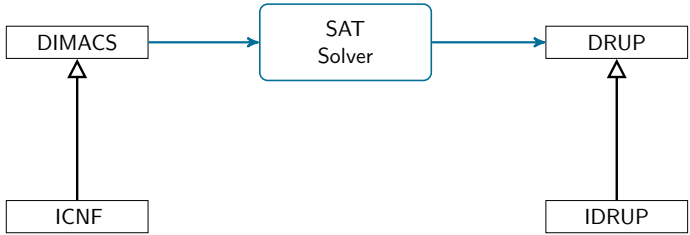

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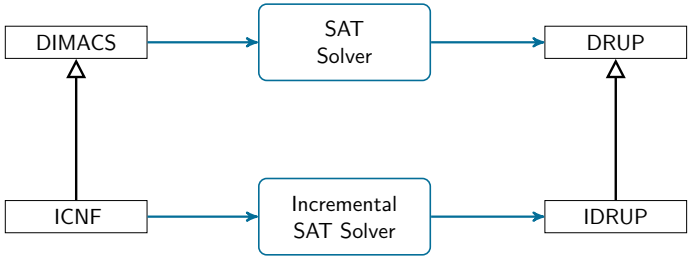

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Syntax & Semantics of the New Formats

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<comments> = { <comment> "\n" }
<lines>    = { <comment> "\n" | <line> "\n" }
<comment>  = "c"   " " <anything-but-new-line>
<line>     = <tag> " " { <literal> " " } "0"
           | "s"   " " <status>
<tag>      = "i" | "q" | "u" | "m"
<status>   = "SATISFIABLE"
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<literal>  = <pos> | <neg>
<pos>      = "1" | "2" | ... | <INT_MAX>
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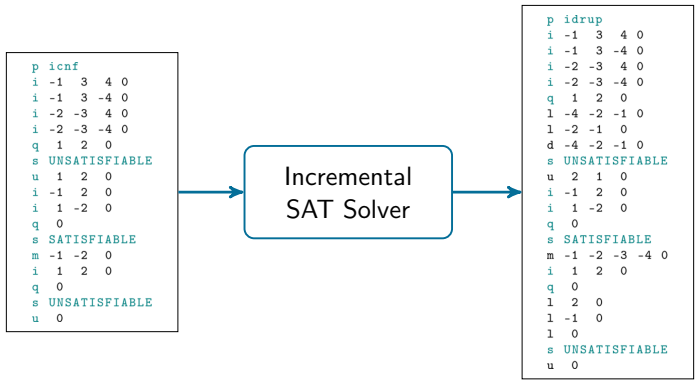
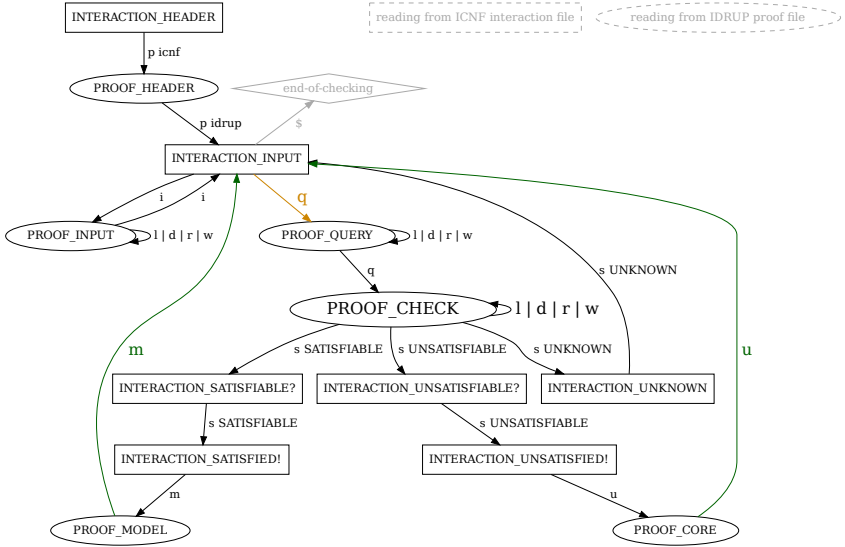
- F_A, F_P: *active* and *passive* clauses

$$(F_A^{i+1}, F_P^{i+1}) = \begin{cases} (F_A^i \cup \{L_i\}, F_P^i) & \text{if } t(L_i) = \texttt{"i"} \\ (F_A^i \cup \{L_i\}, F_P^i) & \text{if } t(L_i) = \texttt{"l"} \\ (F_A^i \setminus \{L_i\}, F_P^i) & \text{if } t(L_i) = \texttt{"d"} \\ (F_A^i \setminus \{L_i\}, F_P^i \cup \{L_i\}) & \text{if } t(L_i) = \texttt{"w"} \\ (F_A^i \cup \{L_i\}, F_P^i \setminus \{L_i\}) & \text{if } t(L_i) = \texttt{"r"} \\ (F_A^i, F_P^i) & \text{otherwise.} \end{cases}$$

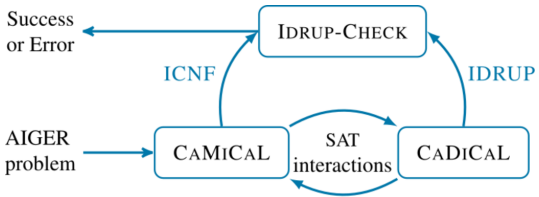
ICNF & IDRUP Example



IDRUP-CHECK – Checking Incremental Proofs

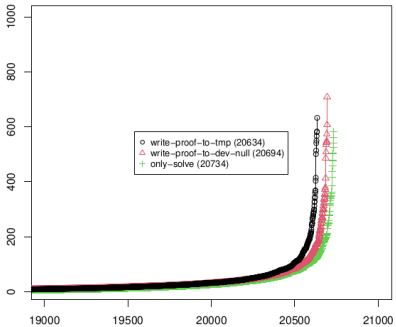


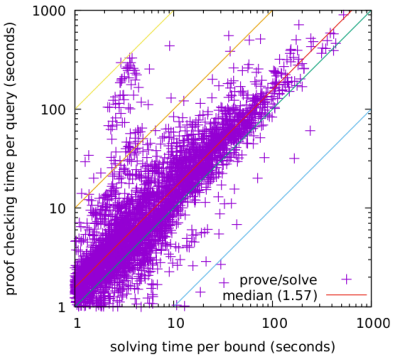
Experiments



- Hardware Model Checking Competition Benchmark set (2017), 300 instances
- Limits: 16 GB memory, 1000 second, maximum bound: k = 100
  - $\rightarrow$ at most 101 incremental SAT query for each instance

Results





- Very small overhead of proof writing
- Reasonable proof checking time ($\sim$ 2x)

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